BLOOM | BMO Bloom Maths Olympiad

GRADE 10

Bloom Maths Olympiad
Sample Paper 1

Maximum Time : 60 Minutes **Maximum Marks : 60**

INSTRUCTIONS

1. There are 50 Multiple Choice Questions in this paper divided into two sections :

   Section A 40 MCQs; 1 Mark each

   Section B 10 MCQs; 2 Marks each

2. Each question has Four Options out of which **ONLY ONE** is correct.
3. All questions are compulsory.
4. There is no negative marking.
5. No electronic device capable of storing and displaying visual information such as calculator and mobile is allowed during the course of the exam.

School Name

Student's Name

Section A (1 Mark Questions)

1. The rationalising factor of $\sqrt[7]{x^2y^3z^5}$ is ......

(a) $\sqrt[7]{y^4z^2x^5}$ (b) $\sqrt[4]{x^3y^2z}$

(c) $\sqrt{x^4y^2z^5}$ (d) $\sqrt[3]{y^2x^4z^3}$

2. Ram, Dwij and Anuj go for a morning walk. They step off together and their steps measure 25 cm, 32 cm and 40 cm, respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?

(a) 1109 cm (b) 800 cm

(c) 1100 cm (d) 1600 cm

3. Read the statements carefully and state 'T' for true and 'F' for false.

(i) $\frac{113}{13}$ is a terminating decimal.

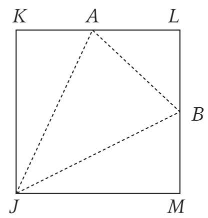
(ii) $\frac{321}{158}$ is a non-terminating decimal.

(iii) $\frac{6805}{9\times3\times5^2}$ is a non-terminating decimal.

(iv) $\frac{7105}{7\times5\times5}$ is a terminating decimal.

	(i)	(ii)	(iii)	(iv)
(a)	F	T	T	T
(b)	T	F	T	T
(c)	T	T	F	F
(d)	F	T	F	T

4. In the given figure *JKLM* is a square with sides of length 8 units. Points *A* and *B* are the mid-points of sides *KL* and *LM* respectively. If a point is selected at random from the interior of the square. What is the probability that the point will be chosen from the interior of ΔALB?



(a) $\frac{5}{8}$ (b) $\frac{7}{8}$ (c) $\frac{3}{4}$ (d) $\frac{1}{8}$

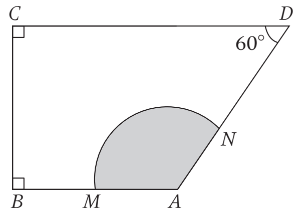
5. A letter is chosen at random from the letters of the word 'CIVILIZATION'. Find the probability that the chosen letter is a vowel.

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{6}{11}$ (d) $\frac{7}{12}$

6. If $\cot\theta = \frac{m}{n}$, then what is the value of $\frac{m\cos\theta - n\sin\theta}{m\cos\theta + n\sin\theta}$?

(a) $\frac{m^2+n^2}{m^2-n^2}$ (b) $\frac{m^2-n^2}{m^2+n^2}$ (c) $\frac{m+n}{m-n}$ (d) $\frac{m-n}{m+n}$

7. In the given figure $AM = 4$ cm, the area of the shaded region is ....



(a) 3π cm^2 (b) $\frac{16\pi}{3}$ cm^2

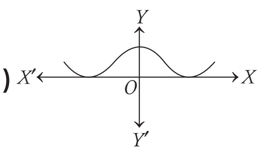
(c) 9π cm^2 (d) $\frac{15\pi}{4}$ cm^2

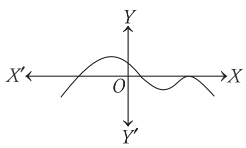
8. There are 35 trees at equal distances of 5 m in a line with a well, the distance of the well from the nearest tree being 10 m. A gardener waters all the trees separately starting from the well and he returns to the well after watering each tree to get water for the next. Find the total distance the gardener will cover in order to water all the trees.

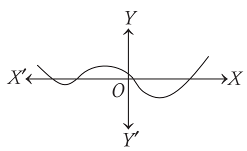
(a) 3000 m (b) 4700 m

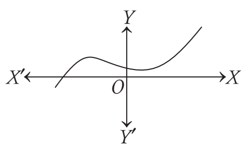
(c) 3500 m (d) 6650 m

9. Which of the following graph has only two distinct real roots?

(a)


(b)


(c)


(d)


10. A rectangular garden of length $(2x^3 + 5x^2 - 6)$ m has the perimeter $(4x^3 - 2x^2 + 8)$ m. Find the breadth of the garden.

(a) $(6x^2 - 9)$ m (b) $(-6x^2 + 10)$ m

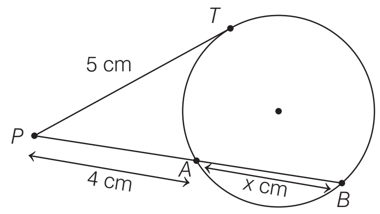
(c) $(2x^3 - 7x^2 + 11)$ m (d) $(6x^3 + 7x^2 + 9)$ m

11. The ratio of a 2-digit number to the sum of digits of that number is 4 : 1. If the digit in the unit place is 4 more than the digit in the tens place, what is the number?

(a) 63 (b) 42 (c) 84 (d) 48

3

12. In the given figure PAB is a secant of circle and PT is the tangent at P of circle. If $PT = 5$ cm, $PA = 4$ cm and $AB = x$ cm, then what will be x?



(a) 5 cm (b) $\frac{9}{4}$ cm (c) $\frac{4}{9}$ cm (d) $\frac{2}{3}$ cm

13. A cylinder is of height 31 cm and base radius 7 cm. A hemisphere of radius equal to base radius of cylinder is cut off from one end and a cone of maximum height from remaining part is also cut off. The curved surface area of the remaining part is

(a) 2222 cm^2 (b) 2508 cm^2 (c) 2510 cm^2 (d) 2212 cm^2

14. Find the mode for the following data.

Age	0-6	6-12	12-18	18-24	24-30	30-36	36-42
Frequency	6	11	25	15	18	12	6

(a) 20.22 (b) 18.5 (c) 15.5 (d) 15.25

15. The denominator of a fraction is one more than the numerator. If one more than the fraction is $2\frac{6}{21}$. Find the fraction.

(a) $\frac{9}{7}$ (b) $\frac{7}{9}$ (c) $\frac{4}{3}$ (d) $\frac{3}{4}$

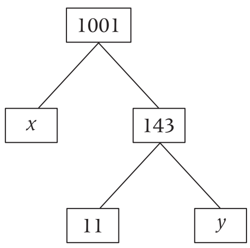
16. In a gift shop, if the shopkeeper displays the gifts in the form of a square then he is left with 36 gifts. If he wanted to increase the size of square by one unit each side of the square he found that 25 gifts fall short of in completing the square. The actual number of gifts which he had with him in the shop was ......

(a) 1690 (b) 936 (c) 538 (d) Can not be determined

17. The ratio of the sum of m and n terms of an AP is $m^2 : n^2$, then find the ratio of mth and nth terms.

(a) $2m-1 : 2n-1$ (b) $2m+1 : 2n+1$ (c) $2m : n$ (d) $m : n$

18. The values of x and y in the given figure are



(a) 7, 13 (b) 13, 7 (c) 9, 12 (d) 12, 9

19. For some integer q, every odd integer is of the form

(a) q (b) $q+1$ (c) $2q$ (d) $2q+1$

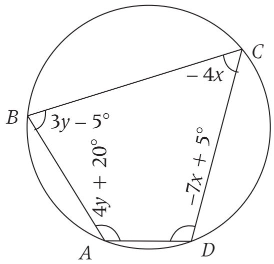
20. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$ respectively, then $g(x)$ is equal to

(a) $x^2 + x + 1$ (b) $x^2 + 1$ (c) $x^2 - x + 1$ (d) $x^2 - 1$

21. In ΔABC, $\angle C = 3\angle B = 2(\angle A + \angle B)$. Then three angles are

(a) 20°, 40° and 60° (b) 20°, 40° and 120° (c) 20°, 40° and 80° (d) 40°, 60° and 120°

22. $ABCD$ is a cyclic quadrilateral. What is the value of $\angle D$ in the cyclic quadrilateral?



(a) 120° (b) 110° (c) 70° (d) 60°

23. A line segment AB is 8 cm in length. AB is produced to P such that $BP^2 = AB \cdot AP$. Then, the length of BP is

(a) $5(\sqrt{5}+1)$ cm (b) $\sqrt{5}+1$ cm (c) $4(\sqrt{5}+1)$ cm (d) $\sqrt{3}+1$ cm

24. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is

(a) 2 (b) -2 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

25. There are two triangles namely RPQ and RST. S and T are points on sides PR and QR of ΔPQR, such that $\angle P = \angle RTS$. The relation between the two triangles is that,

(a) $\Delta RPQ \cong \Delta RST$ (b) $\Delta RPQ \sim \Delta RTS$
(c) $\Delta RPQ \cong \Delta RST$ (d) $\Delta RPQ = \Delta RST$

26. Determine the AP whose 3rd term is 16 and the 7th term exceeds the 5th term by 12.

(a) 1, 3, 5, 7, ... (b) 4, 12, 20, 28, 32, ...
(c) 6, 9, 11, 14, ... (d) 4, 10, 16, 22, ...

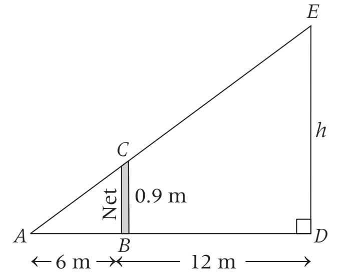
27. A vertical pole of length 6m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower (in m).

(a) 42 (b) 48 (c) 36.5 (d) 52.5

28. How many numbers of two digits are divisible by 7?

(a) 11 (b) 13 (c) 21 (d) 17

29. Find the value of the height 'h' in the adjoining figure, at which the tennis ball must be hit, so that it will just pass over the net and land 6 m away from the base of the net.



(a) 2.7 m (b) 3.8 m (c) 6.9 m (d) 1.8 m

30. For what value of n, the nth terms of the AP's 63, 65, 67, ... and 3, 10, 17, ... are equal?

(a) 13 (b) 14 (c) 18 (d) 22

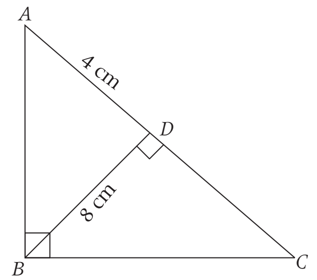
31. Find the sum given below $7 + 10\frac{1}{2} + 14 + \dots + 84$

(a) $501\frac{1}{2}$ (b) $1362\frac{6}{7}$ (c) $1046\frac{1}{2}$ (d) 1272

32. Find the area of the ΔABC whose vertices are $A(1, 1)$, $B(12, 2)$ and $C(7, 21)$.

(a) 140 sq unit (b) 150 sq unit (c) 132 sq unit (d) 107 sq unit

33. In the given figure, $\angle ABC = 90^\circ$ and $BD \perp AC$. If $BD = 8$ cm and $AD = 4$ cm, then find the value of CD.



(a) 12 cm (b) 16 cm (c) 8 cm (d) 4 cm

34. The value of $\dfrac{2^{7m+4} \times 3^{2m-3n} \times 5^{5m+3n+4} \times 6^{m+2n-3}}{10^{2m+4n+7}\ 15^{3m-n-3}\ 2^{6m-2n-5}}$ is

(a) 1 (b) 0 (c) 2 (d) $\frac{1}{2}$

35. Coordinates of P and Q are $(4, -2)$ and $(-1, 7)$. The abscissa of a point R on the line segment PQ such that $\frac{PR}{PQ} = \frac{3}{5}$ is

(a) $\frac{18}{5}$ (b) $\frac{17}{5}$ (c) 1 (d) $\frac{17}{8}$

36. D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the areas of ΔDEF and ΔABC.

(a) 1 : 2 (b) 1 : 4 (c) 2 : 3 (d) $2 : \frac{1}{\sqrt{2}}$

37. If a cubic polynomial with the sum of its zeroes, sum of the products of zeroes taken two at a time and product of its zeroes as 0, – 7 and – 6 respectively, then the cubic polynomial is

(a) $x^3 + 7x - 6$ (b) $x^3 + 7x + 6$ (c) $x^3 - 7x - 6$ (d) $x^3 - 7x + 6$

38. If the zeroes of the polynomial $ax^2 + bx + b$ are in the ratio $m : n$, then find the value of $\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}}$.

(a) $\frac{b}{a}$ (b) $-\sqrt{\frac{b}{a}}$ (c) 1 (d) $\sqrt{\frac{b}{a}}$

39. A class teacher says to three students Sanjeev, Anjali and Paras for making greeting cards. Each person take time 15 min, 20 min and 25 min respectively for making these cards.

(i) If all of them making card together, then after what time they will prepare a new card together?

(ii) Suppose if they start working at same time. For how much time they work together?

	(i)	(ii)
(a)	300 min	15 min
(b)	300 min	20 min
(c)	200 min	10 min
(d)	100 min	15 min

40. The angry Arjun carried some arrows for fighting with Bheeshm. With half the arrows, he cut down the arrows thrown by Bheeshm on him and with six other arrows he killed the charioteer of Bheeshm. With one arrow each he knocked down respectively the rath, flag and bow of Bheeshm. Finally, with one more than four times the square root of arrows he laid Bheeshm unconcious on an arrow-bed. Find the total number of arrows Arjun had.

(a) 150 (b) 200 (c) 120 (d) 100

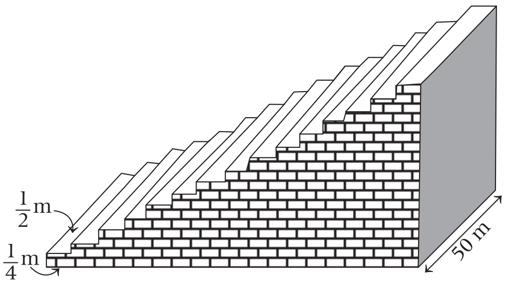
Section B (2 Marks Questions)

41. The vertices of a ΔABC are $A(4, 6), B(1, 5)$ and $C(7, 2)$. A line is drawn to intersect sides AB and AC at D and E respectively, such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Calculate the area of the ΔADE.

(a) $\frac{15}{2}$ sq units (b) $\frac{15}{32}$ sq units

(c) $\frac{13}{10}$ sq units (d) $\frac{10}{13}$ sq units

42. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete. Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (see figure). Calculate the total volume of concrete (in m^3) required to build the terrace.



(a) 625 (b) 860 (c) 750 (d) 1000

43. A sum of ₹ 2000 is invested at 7% simple interest per year. Then find the interest at the end of 20th yr making use of this fact.

(a) ₹ 2600 (b) ₹ 3200 (c) ₹ 2800 (d) ₹ 3000

44. $(\sec^2 A - 1) + \left(1 + \dfrac{1}{\tan^2 A}\right)$ is equal to

(a) $\dfrac{1}{\sin^2 A - \sin^4 A}$ (b) $1 + \dfrac{1}{\sin^2 A - \sin^4 A}$

(c) $\dfrac{\cos^2 A}{\sin A + \sin^2 A}$ (d) $-1 + \dfrac{1}{\sin^2 A - \sin^4 A}$

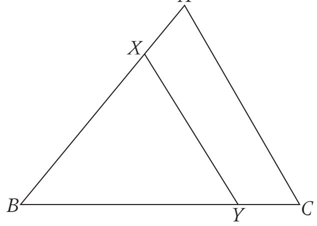
45. The value of n for which the expression $x^4 + 4x^3 + nx^2 + 4x + 1$ becomes perfect square is

(a) 3 (b) 4 (c) 5 (d) 6

46. A building which is 30 m high was observed from a point on the ground. Observer found the angle of elevation of a point on the second floor of the building which is 10 m above the ground same as the angle subtended by the rest of the building above the point P. If the height of the observer is to be ignored, approximate distance between the observer and the foot of the building is (take $\sqrt{3} = 1.732$) and $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

(a) 17.32 m (b) 20 m (c) 21.21 m (d) None of these

47. In the given figure, the line segment XY is parallel to side AC of ΔABC and it divides the triangle into two parts of equal area. Then, find



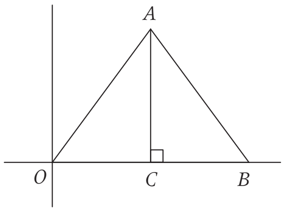
(i) $AX : AB$ (ii) $\dfrac{AC}{XY}$

	(i)	(ii)
(a)	$(2+\sqrt{2}):2$	$\sqrt{2}-2$
(b)	$(2-\sqrt{2}):2$	$\sqrt{2}$
(c)	$(2-\sqrt{3}):3$	3
(d)	$(2+\sqrt{2}):3$	$\sqrt{2}-3$

48. The internal and external diameters of a hollow hemispherical vessel are 24 cm and 25 cm respectively. The cost to paint 1 cm^2 of the surface is ₹ 0.5. Find the total cost of painting the vessel all over.

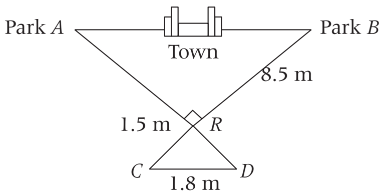
(a) ₹ 832 (b) ₹ 996.28 (c) ₹ 1001.59 (d) ₹ 962.89

49. If the area of the equilateral ΔOAB shown in figure is $9\sqrt{3}$ sq units, then what are the coordinates of point A?



(a) $(3, \sqrt{3})$ (b) $(3, \sqrt{3}/2)$ (c) $(3, 3\sqrt{3})$ (d) $(2, \sqrt{3})$

50. Mason Construction wants to connect two parks on opposite sides of town with a road. Surveyors have laid out a map as shown. The road can be built through the town or around town through point R. The roads intersect at a right angle at point R. The line joining Park A to Park B is parallel to the line joining C and D.



(i) What is the distance between the parks through town?

(ii) What is the distance from Park A to Park B through point R?

	(i)	(ii)
(a)	10.2 m	14.11 m
(b)	10 m	12.5 m
(c)	8.75 m	12 m
(d)	12 m	13.33 m

Solutions

1. (a) Rationalising factor is a term with which a term is multiplied or divided to make the whole term rational.

$$\because \sqrt[7]{x^2y^3z^5} \times \sqrt[7]{y^4z^2x^5} = \sqrt[7]{x^2y^3z^5 \times x^5y^4z^2}$$

$$= (x^{2+5} \times y^{3+4} \times z^{5+2})^{\frac{1}{7}} \qquad [\because a^m \times a^n = a^{m+n}]$$

$$= (x^7 \times y^7 \times z^7)^{\frac{1}{7}}$$

$$= (xyz)^{\frac{7}{7}} \qquad [\because (a^m)^{\frac{1}{n}} = a^{\frac{m}{n}}]$$

$$= xyz$$

$\therefore$ Rationalising factor of $\sqrt[7]{x^2y^3z^5}$ is $\sqrt[7]{y^4z^2x^5}$.

2. (b) To find the minimum distance that each should walk and cover the same distance is equal to

LCM of 25 cm, 32 cm and 40 cm

2	25, 32, 40
2	25, 16, 20
2	25, 8, 10
2	25, 4, 5
2	25, 2, 5
5	25, 1, 5
5	5, 1, 1,
	1, 1, 1

$\therefore$ LCM of (25, 32, 40) $= 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 = 2^5 \times 5^2 = 800$ cm

So, each should walk 800 cm so, that each can cover the same distance in complete steps.

3. (a) (i) $\frac{113}{13} = 8.692307$ is a non-terminating repeating decimal.

(ii) $\frac{321}{158} = 2.03164557$ is a non-terminating decimal.

(iii) $\frac{6805}{27 \times 5^2} = \frac{6805}{27 \times 25} = \frac{6805}{675} = 10.08145...$ is a non-terminating decimal.

(iv) $\frac{7105}{7 \times 5 \times 5} = \frac{1015}{5 \times 5} \times \frac{2 \times 2}{2 \times 2} = \frac{4060}{100} = 40.6$

It is a terminating decimal.

4. (d) Area of square $JMLK = 8^2 = 64$ sq units

A and B are the mid-points of sides KL and LM.

$\therefore \quad AL = KA = LB = BM = 4$ units

Now, Area of $\Delta ALB = \frac{1}{2} \times AL \times LB$

$= \frac{1}{2} \times 4 \times 4 = \frac{16}{2}$ sq units $= 8$ sq units

$\therefore$ Required probability $= \frac{8}{64} = \frac{1}{8}$

5. (a) Total number of letters in 'CIVILIZATION' $= 12$

Vowels are I, I, I, A, I, O i.e. 6 vowels.

$\therefore$ Probability of getting a vowel $= \frac{6}{12} = \frac{1}{2}$

6. (b) Given, $\cot\theta = \frac{m}{n}$

Now, $\quad \frac{m\cos\theta - n\sin\theta}{m\cos\theta + n\sin\theta} = \frac{m\cot\theta - n}{m\cot\theta + n}$ [$\because$ on dividing by $\sin\theta$]

$= \frac{m \times \frac{m}{n} - n}{m \times \frac{m}{n} + n}$ [$\because$ put the value of $\cot\theta$]

$= \frac{m^2 - n^2}{m^2 + n^2}$

7. (b) In quadrilateral $ABCD$,

$\angle A + \angle B + \angle C + \angle D = 360°$ [sum of angle of a quadrilateral is $360°$]

$\angle A + 90° + 90° + 60° = 360°$

$\angle A = 360° - 240° = 120°$

$\therefore$ Area of sector AMN = Area of shaded region

$= \frac{\pi r^2 \theta}{360} = \frac{\pi \times (4)^2 \times 120°}{360°} = \frac{16\pi}{3}$ cm^2

8. (d) Since, distance of nearest tree from the well $= 10$ m

Also, each tree is at equal distance of 5 m from the next tree.

$\therefore$ AP formed is 10, 15, 20, ...

Here, $\quad a = 10, d = 5$ and $n = 35$

$\therefore \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad S_{35} = \frac{35}{2}[2(10) + (35-1)5] = \frac{35}{2}[20 + 34 \times 5] = \frac{35}{2}[20 + 170] = \frac{35}{2} \times 190 = 3325$

Hence, the total distance the gardener will cover in order to water all the trees

$$= 2 \times 3325 = 6650 \text{ m}$$

[here, we multiply by 2, because in each time gardner watering the tree, which is taken from well]

9. (a) For two distinct real roots, the graph must cut or touch X-axis only two times. So, graph in option (a) has only two distinct real roots.

10. (b) We know that, perimeter of rectangle = 2 (length + breadth)

$$\therefore \quad 2[(2x^3 + 5x^2 - 6) + \text{breadth}] = (4x^3 - 2x^2 + 8)$$

$$2[2x^3 + 5x^2 - 6 + \text{breadth}] = 2[2x^3 - x^2 + 4]$$

$\therefore$ Breadth $= 2x^3 - x^2 + 4 - 2x^3 - 5x^2 + 6 = (-6x^2 + 10)$ m

11. (d) Let the digits at unit place and tens place be y and x respectively.

$\therefore$ The number $= 10x + y$

According to the question, $\dfrac{10x + y}{x + y} = \dfrac{4}{1}$

$$\Rightarrow \quad 10x + y = 4x + 4y$$

$$\Rightarrow \quad 6x = 3y$$

$$\Rightarrow \quad y = 2x \qquad \text{...(i)}$$

Also given the digit in the unit place is 4 more than the digit in the tens place.

i.e., $\quad y = x + 4 \Rightarrow 2x = x + 4 \quad$ [$\because$ from Eq. (i)]

$$x = 4$$

Then, $\quad y = 2x = 2 \times 4 = 8$

$\therefore \quad$ Number $= 10x + y = 10 \times 4 + 8 = 48$

12. (b) By the theorem of chord and tangent of circle.

$$PT^2 = PA \times PB$$

$$\Rightarrow \quad (5)^2 = 4 \times (4 + x) \Rightarrow 25 = 16 + 4x$$

$$\Rightarrow \quad 4x = 9$$

$$\Rightarrow \quad x = \frac{9}{4} \text{ cm}$$

13. (a) Curved surface area of the remaining solid

$$= \text{Curved surface area of [cylinder + cone + sphere]}$$

$$= 2\pi rh + \pi rl + 2\pi r^2$$

$$= \left(2 \times \frac{22}{7} \times 7 \times 31\right) + \left(\frac{22}{7} \times 7 \times \sqrt{7^2 + (31-7)^2}\right) + \left(2 \times \frac{22}{7} \times 7^2\right)$$

$$= 44 \times 31 + 22 \times 25 + 44 \times 7$$

$$[\because l = \sqrt{7^2 + (31-7)^2} = \sqrt{49 + 576} = \sqrt{625} = 25]$$

$$= 1364 + 550 + 308 = 2222 \text{ cm}^2$$

14. (c) Since, maximum class frequency is 25, so the modal class is 12-18.

$\therefore \quad l = 12, f_1 = 25, f_0 = 11, f_2 = 15, h = 6$

$$\therefore \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 12 + \left(\frac{25 - 11}{2 \times 25 - 11 - 15}\right) \times 6 = 12 + \frac{14 \times 6}{24} = 12 + 3.5 = 15.5$$

15. (a) Let the numerator of the fraction be x.

Then, denominator of the fraction $= x + 1$

$\therefore$ Fraction $= \dfrac{x}{x+1}$.

According to the question,

$$\frac{x}{x+1} + 1 = 2\frac{6}{21}$$

$$\Rightarrow \quad \frac{x}{x+1} = \frac{48}{21} - 1 = \frac{48 - 21}{21}$$

$$\Rightarrow \quad \frac{x}{x+1} = \frac{27}{21} = \frac{9}{7}$$

16. (b) Let the number of gifts in a side of square $= x$

According to the question,

$$x^2 + 36 = \text{Total number of gifts} \quad \text{...(i)}$$

Also, $\quad (x+1)^2 - 25 = \text{Total number of gifts} \quad$ ...(ii)

From Eqs. (i) and (ii), we have

$$x^2 + 36 = (x+1)^2 - 25 \Rightarrow x^2 + 36 = x^2 + 2x + 1 - 25$$

$$\Rightarrow \quad 36 + 24 = 2x \Rightarrow 60 = 2x \Rightarrow x = 30$$

$\therefore$ Total number of gifts $= (30)^2 + 36 = 900 + 36 = 936$

17. (a) Let first term and common ratio of AP be a and d respectively.

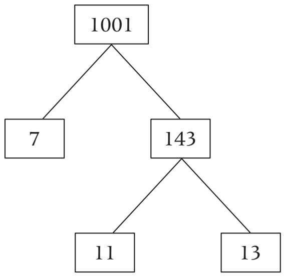
Given, $\dfrac{S_m}{S_n} = \dfrac{m^2}{n^2} \Rightarrow \dfrac{\frac{m}{2}[2a + (m-1)d]}{\frac{n}{2}[2a + (n-1)d]} = \dfrac{m^2}{n^2} \Rightarrow \dfrac{2a + (m-1)d}{2a + (n-1)d} = \dfrac{m}{n}$

Now, replace m with $2m - 1$ and n with $2n - 1$, we get

$$\frac{2a + 2(m-1)d}{2a + 2(n-1)d} = \frac{2m-1}{2n-1} \Rightarrow \frac{a + (m-1)d}{a + (n-1)d} = \frac{2m-1}{2n-1} \Rightarrow \frac{a_m}{a_n} = \frac{2m-1}{2n-1} = 2m - 1 : 2n - 1$$

18. (a) Given, number is 1001.

Then, the factor tree of 1001 is given as below



$\therefore \quad 1001 = 7 \times 11 \times 13$

By comparing with given factor tree, we get

$$x = 7,\ y = 13$$

19. (d) If q is any integer, then $2q+1$ is always odd integer.

e.g. Suppose $q = 3$; $2q + 1 = 2 \times 3 + 1 = 7$ (odd)

Suppose $q = 4$; $2q + 1 = 2 \times 4 + 1 = 9$ (odd)

20. (c) Here, dividend $= x^3 - 3x^2 + x + 2$,

Quotient $= x - 2$, Remainder $= -2x + 4$

Since, dividend = Divisor $\times$ Quotient + Remainder

So, $\quad x^3 - 3x^2 + x + 2 = g(x) \times (x-2) + (-2x+4)$

$\Rightarrow \quad g(x) \times (x-2) = x^3 - 3x^2 + x + 2 + 2x - 4$

$$\Rightarrow \quad g(x) = \frac{x^3 - 3x^2 + 3x - 2}{x-2}$$

$$= \frac{(x-2)(x^2 - x + 1)}{(x-2)} = x^2 - x + 1$$

21. (b) Given, $\angle C = 3\angle B$ ...(i)

and $\quad \angle C = 2(\angle A + \angle B)$ ...(ii)

Since, the sum of the three angles of a triangle is 180°.

$\therefore \quad \angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \quad \angle A + \angle B + 2(\angle A + \angle B) = 180^\circ$ [from Eq. (ii)]

$\Rightarrow \quad 3\angle A + 3\angle B = 180^\circ$

$\Rightarrow \quad \angle A + \angle B = 60^\circ$ [dividing by 3] ...(iii)

Again, $\quad \angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \quad 60^\circ + 3\angle B = 180^\circ$ [from Eq. (i) and (ii)]

$\Rightarrow \quad 3\angle B = 120^\circ$

$\Rightarrow \quad \angle B = 40^\circ$

On putting $\angle B = 40^\circ$ in Eq. (iii), we get

$$\angle A + 40^\circ = 60^\circ$$

$\Rightarrow \quad \angle A = 20^\circ$

On putting $\angle B = 40^\circ$ in Eq. (i), we get

$$\angle C = 3 \times 40^\circ = 120^\circ$$

Hence, angles are $\angle A = 20^\circ$, $\angle B = 40^\circ$ and $\angle C = 120^\circ$.

22. (b) We know that, in a cyclic quadrilateral, the sum of two opposite angles is 180°.

$\therefore \quad \angle B + \angle D = 180^\circ$ and $\angle A + \angle C = 180^\circ$

$\Rightarrow \quad 3y - 5^\circ - 7x + 5^\circ = 180$ and $4y + 20^\circ - 4x = 180^\circ$

$\Rightarrow \quad 3y - 7x = 180^\circ \quad$ ...(i)

and $\quad 4y - 4x = 160^\circ$

$\Rightarrow \quad y - x = 40^\circ \quad$ [dividing both sides by 4] ...(ii)

On multiplying Eq. (ii) by 7 and then subtracting Eq. (i), we get

$$-4y = 180^\circ - 280^\circ$$

$\Rightarrow \quad -4y = -100^\circ$

$\Rightarrow \quad y = 25^\circ$

On putting $y = 25^\circ$ in Eq. (ii), we get

$$25^\circ - x = 40^\circ$$

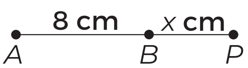
$\Rightarrow \quad x = -15^\circ$

$\therefore \quad \angle D = -7x + 5^\circ = -7 \times (-15^\circ) + 5^\circ$

$$= 105^\circ + 5^\circ = 110^\circ$$

23. (c) Let $BP = x$ cm.

Then, $AP = AB + BP = (8 + x)$ cm.



Given, $BP^2 = AB \cdot AP \Rightarrow x^2 = 8 \cdot (8 + x)$

$\Rightarrow \quad x^2 - 8x - 64 = 0$

$\therefore \quad x = \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot (-64)}}{2}$

or $\quad x = \dfrac{8 \pm \sqrt{64 \times 5}}{2}$

$$= \frac{8 \pm 8\sqrt{5}}{2}$$

$$= 4 \pm 4\sqrt{5}$$

But the length of BP is positive.

So, $x = (4 + 4\sqrt{5})$ cm $= 4(\sqrt{5} + 1)$ cm

24. (a) Given equation is, $x^2 + kx - \dfrac{5}{4} = 0$ and $x = \dfrac{1}{2}$ is a root of the equation.

So, it satisfies the given equation.

$\therefore \quad \left(\dfrac{1}{2}\right)^2 + k \times \left(\dfrac{1}{2}\right) - \dfrac{5}{4} = 0$

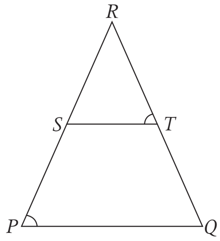
$$\Rightarrow \quad \frac{1}{4}+\frac{k}{2}-\frac{5}{4}=0$$

$$\Rightarrow \quad \frac{k}{2}=\frac{5}{4}-\frac{1}{4}=\frac{5-1}{4}$$

$$\Rightarrow \quad \frac{k}{2}=\frac{4}{4}=1$$

$$\therefore \quad k=2\times 1=2$$

25. (b) Draw ΔPQR, such that S and T are points on sides PR and QR respectively. We join the points S and T.



From the above figure, we have ΔRPQ and ΔRTS in which

$\angle RPQ = \angle RTS$ [given]

and $\angle PRQ = \angle SRT$ [common angle]

$\therefore$ $\Delta RPQ \sim \Delta RTS$ [by AA similarity criterion]

26. (d) Let a be the first term and d be the common difference of given AP.

Given that, the third term of the AP is

$$a_3 = 16$$

$\Rightarrow$ $a+2d=16$ $[\because a_n = a+(n-1)d]$ ...(i)

Also, it is given that

7th term of an AP $=12+$ 5th term of an AP, i.e.

$$a_7 = 12 + a_5$$

$$\Rightarrow \quad a_7 - a_5 = 12$$

$$\Rightarrow \quad (a+6d)-(a+4d)=12$$

$$\Rightarrow \quad 2d = 12 \Rightarrow d = 6$$

On putting $d=6$ in Eq. (i), we get

$$a+2\times 6=16 \Rightarrow a=16-12=4$$

We know that, general form of an AP is

$$a,\ a+d,\ a+2d,\ a+3d,\ \ldots.$$

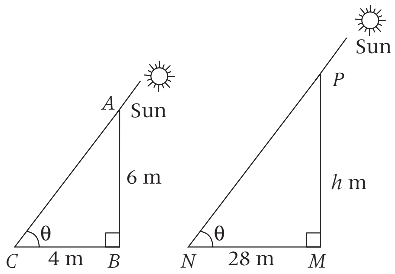
Then, the required AP is

$$4,\ 4+6,\ 4+2\times 6,\ 4+3\times 6,\ \ldots,$$

i.e., 4, 10, 16, 22, ...

27. (a) Let AB be a pole of length 6m, $BC = 4$ m be the length of its shadow and θ be the angle which Sun makes with ground. Also, at the same time, another tower casts shadow NM of length 28 m.

Let $PM = h$ m be the height of the tower and θ be the angle which Sun ray makes with ground.



In ΔABC and ΔPMN,

$$\angle ABC = \angle PMN \quad \text{[each } 90^\circ\text{]}$$

and $$\angle ACB = \angle PNM \quad \text{[each } \theta^\circ\text{]}$$

$\therefore$ $$\Delta ABC \sim \Delta PMN \quad \text{[by AA similarity criterion]}$$

Now, $$\frac{AB}{PM} = \frac{BC}{MN}$$

[since, corresponding sides of similar triangles are proportional]

$\Rightarrow$ $$\frac{AB}{BC} = \frac{PM}{MN} \Rightarrow \frac{6}{4} = \frac{h}{28} \quad [\because AP = 6 \text{ m}, BC = 4 \text{ m and } MN = 28 \text{ m}]$$

$\Rightarrow$ $$h = \frac{6 \times 28}{4} = 42 \text{ m}$$

Hence, the height of the tower is 42 m.

28. (b) Two-digits numbers are 10, 11, 12, 13, 14, 15, ..., 97, 98, 99 in which only 14, 21, 28, ..., 98 are divisible by 7.

Here, $$21 - 14 = 28 - 21 \ldots = 7$$

So, this list of numbers forms an AP, whose first term $(a) = 14$, common difference $(d) = 7$.

Let there are n terms in the above sequence, then $a_n = 98$

$\Rightarrow$ $$a + (n-1)d = 98 \quad [\because a_n = a + (n-1)d]$$

$\Rightarrow$ $$14 + (n-1)7 = 98 \Rightarrow 14 + 7n - 7 = 98$$

$\Rightarrow$ $$7n = 91 \Rightarrow n = \frac{91}{7} = 13$$

Hence, 13 numbers of two digits are divisible by 7.

29. (a) Since, the height h is measured vertically, so $\angle EDA$ is a right angle. We assume that the net (i.e. CB) is vertical.

Here, ΔADE and ΔABC are similar [by AA similarity criterion]

$\therefore$ $$\frac{DE}{BC} = \frac{AD}{AB}$$ [since, corresponding sides of similar triangles are proportional]

$\Rightarrow$ $$\frac{h}{0.9} = \frac{18}{6} \Rightarrow \frac{h}{0.9} = 3 \Rightarrow h = 0.9 \times 3 = 2.7 \text{ m}$$

Hence, the height at which the ball should be hit, is 2.7 m.

30. (a) Given, AP's are 63, 65, 67, ... and 3, 10, 17, ...

Here, first term of first AP $(a_1) = 63$

common difference of first AP $(d_1) = 65 - 63 = 2$

first term of second AP $(a_2) = 3$

and common difference of second AP $(d_2) = 10 - 3 = 7$

According to the question, nth term of both AP's are equal.

$\therefore \quad 63 + (n-1)2 = 3 + (n-1)7 \quad [\because a_n = a + (n+1)d]$

$\Rightarrow \quad 7(n-1) - 2(n-1) = 63 - 3$

$\Rightarrow \quad (n-1)(7-2) = 60$

$\Rightarrow \quad (n-1) = \frac{60}{5} = 12$

$\Rightarrow \quad n = 12 + 1 = 13$

Hence, the 13th term of the two given AP's are equal.

31. (c) The given numbers are $7, 10\frac{1}{2}, 14, \ldots, 84$

$\because \quad 10\frac{1}{2} - 7 = 14 - 10\frac{1}{2} = \ldots = \frac{7}{2}$

$\therefore$ The given numbers form an AP.

Here, first term, $a = 7$,

common difference, $\quad d = 10\frac{1}{2} - 7 = 3\frac{1}{2} = \frac{7}{2}$

and last term, $l = a_n = 84$

$\because \quad a_n = a + (n-1)d$

$\therefore \quad 84 = 7 + (n-1)\frac{7}{2} \quad \left[\because a = 7 \text{ and } d = \frac{7}{2}\right]$

$\Rightarrow \quad n - 1 = 22 \Rightarrow n = 23$

$\because$ Sum of n terms of an AP,

$$S_n = \frac{n}{2}(a + l)$$

$\therefore$ Sum of 23 terms, $(S_{23}) = \frac{23}{2}(7 + 84) = \frac{23}{2} \times 91 = \frac{2093}{2} = 1046\frac{1}{2}$

32. (d) Given, vertices of ΔABC are $A(1, 1)$, $B(12, 2)$ and $C(7, 21)$.

For ΔABC, $x_1 = 1$, $y_1 = 1$, $x_2 = 12$, $y_2 = 2$ and $x_3 = 7$, $y_3 = 21$

Area of $\Delta ABC = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$= \frac{1}{2}|1(2 - 21) + 12(21 - 1) + 7(1 - 2)|$

$= \frac{1}{2}|-19 + 240 - 7| = \frac{1}{2}|214|$

$= 107$ sq units

33. (b) Given, $BD = 8$ cm and $AD = 4$ cm

In ΔADB and ΔBDC, $\angle BDA = \angle CDB$ [each 90°]

$\angle DBA = \angle DCB$ [each $(90^\circ - \angle A)$]

$\therefore$ $\Delta ADB \sim \Delta BDC$ [by AA similarity criterion]

$\Rightarrow$ $\dfrac{AD}{BD} = \dfrac{BD}{CD}$

[since, corresponding sides of similar triangles are proportional]

$\Rightarrow$ $CD = \dfrac{BD^2}{AD}$

$\therefore$ $CD = \dfrac{8^2}{4}$

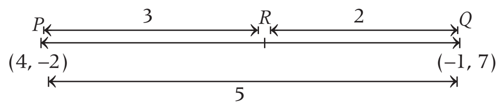
$= \dfrac{64}{4} = 16$ cm

34. (d) $\dfrac{2^{7m+4} \times 3^{2m-3n} \times 5^{5m+3n+4} \times 6^{m+2n-3}}{10^{2m+4n+7} \times 15^{3m-n-3} \times 2^{6m-2n-5}}$

$$= \frac{2^{7m+4} \times 3^{2m-3n} \times 5^{5m+3n+4} \times (2 \times 3)^{m+2n-3}}{(5 \times 2)^{2m+4n+7} \times (3 \times 5)^{3m-n-3} \times 2^{6m-2n-5}}$$

$$= \frac{2^{8m+2n+1} \times 3^{3m-n-3} \times 5^{5m+3n+4}}{2^{8m+2n+2} \times 3^{3m-n-3} \times 5^{5m+3n+4}} = \frac{1}{2}$$

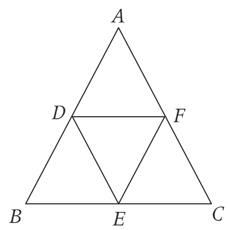
35. (c) Given, points $P(4, -2)$ and $Q(-1, 7)$



and $\dfrac{PR}{RQ} = \dfrac{m}{n} = \dfrac{3}{2}$

$\therefore$ Point of the abscissa $= \dfrac{mx_2 + nx_1}{m+n} = \dfrac{3 \times (-1) + 2 \times (4)}{3+2} = \dfrac{-3+8}{5} = \dfrac{5}{5} = 1$

36. (b) Draw a ΔABC and D, E and F are the mid-points of sides AB, BC and CA, respectively. Join the points D, E and F.



By mid-point theorem, $DF = \dfrac{1}{2} BC$

$$DE = \frac{1}{2} CA \text{ and } EF = \frac{1}{2} AB \quad \text{...(i)}$$

In ΔDEF and ΔCAB, $\frac{DF}{BC} = \frac{DE}{CA} = \frac{EF}{AB} = \frac{1}{2}$ [from Eq. (i)]

$\therefore$ $\Delta DEF \sim \Delta CAB$

Now, $$\frac{\text{ar}(\Delta DEF)}{\text{ar}(\Delta CAB)} = \frac{DE^2}{CA^2} = \frac{\left(\frac{1}{2} CA\right)^2}{CA^2} = \frac{1}{4}$$

$\therefore$ $\text{ar}(\Delta DEF) : \text{ar}(\Delta ABC) = 1 : 4$

37. (d) Let α, β, γ be the zeroes of the required polynomials

$$\alpha + \beta + \gamma = 0$$
$$\alpha\beta + \beta\gamma + \alpha\gamma = -7$$
$$\alpha\beta\gamma = -6$$

$\therefore$ Required cubic polynomial is

$= k[x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \alpha\gamma)x - \alpha\beta\gamma]$ where, k is non-zero constant.

$= k[x^3 - (0)x^2 + (-7)x - (-6)]$

$= x^3 - 7x + 6$ [consider, $k = 1$]

38. (b) Let the zeroes of the given polynomial $ax^2 + bx + b$ be $m\alpha$ and $n\alpha$.

$\therefore$ Sum of zeroes $= -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -\frac{b}{a}$

$$m\alpha + n\alpha = -\frac{b}{a} \Rightarrow m + n = -\frac{b}{a \times \alpha} \quad \text{...(i)}$$

and product of zeroes, $m\alpha \times n\alpha = \frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{b}{a}$

$$m \times n = \frac{b}{a \times \alpha^2} \quad \text{...(ii)}$$

Now, $$\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} = \frac{m+n}{\sqrt{mn}} = -\frac{b}{a \times \alpha} \times \sqrt{\frac{a\alpha^2}{b}}$$ [$\because$ from Eq. (i) and (ii)]

$\therefore$ $$\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} = -\sqrt{\frac{b}{a}}$$

39. (a) (i) The required number of min after which they start preparing a new card together

= LCM of (15, 20, 25)

2	15, 20, 25
2	15, 10, 25
3	15, 5, 25
5	5, 5, 25
5	1, 1, 5
	1, 1, 1

$$= 2 \times 2 \times 3 \times 5 \times 5 = 300 \text{ min}$$

(ii) In all three of the students, Sanjeev take minimum 15 min to complete the greeting card. Hence, minimum 15 min they work together.

40. (d) Let total number of arrows be x.

According to the question, $x - \left(\frac{x}{2} + 6 + 3\right) = 4\sqrt{x} + 1$

$\Rightarrow \quad \frac{x}{2} - 9 = 4\sqrt{x} + 1$

$\Rightarrow \quad \frac{x}{2} - 10 = 4\sqrt{x}$ ... (i)

On squaring both sides, $\frac{x^2}{4} + 100 - 10x = 16x$

$\Rightarrow \quad x^2 - 104x + 400 = 0$

$\Rightarrow \quad x^2 - 100x - 4x + 400 = 0$

$\Rightarrow \quad x(x - 100) - 4(x - 100) = 0$

$\Rightarrow \quad (x - 100)(x - 4) = 0$

$\therefore \quad x = 100$ or $x = 4$

$\because \quad x$ can not be 4, because it does not satisfy the Eq. (i)

$\therefore \quad$ Total number of arrows $= 100$

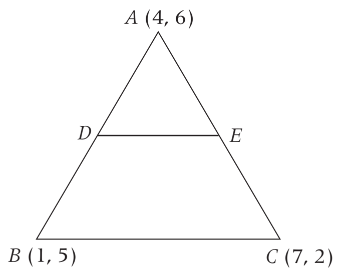
41. (b) Given, $A(4, 6)$, $B(1, 5)$, $C(7, 2)$, therefore

$$A(4, 6) = (x_1, y_1), B(1, 5) = (x_2, y_2), C(7, 2) = (x_3, y_3)$$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

$$= \frac{1}{2}\{4(5-2) + 1(2-6) + 7(6-5)\} = \frac{1}{2}(12 - 4 + 7) = \frac{15}{2}$$

According to the question, the line DE intersect the sides AB and AC at D and E respectively.



and $\quad \frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$

$\therefore \quad \Delta ADE \sim ABC$

By the area property of similar triangles.

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta ABC} = \frac{AD^2}{AB^2}$$

$$\Rightarrow \qquad \text{Area of } \Delta ADE = \left(\frac{AD}{AB}\right)^2 \times \text{Area of } \Delta ABC$$

$$= \frac{1}{16} \times \frac{15}{2} = \frac{15}{32} \text{ sq units.}$$

42. (c) Clearly, volume of concrete required to build the I step, II step, III step, ... are respectively,

$$\frac{1}{4} \times \frac{1}{2} \times 50, \left(2 \times \frac{1}{4}\right) \times \frac{1}{2} \times 50, \left(3 \times \frac{1}{4}\right) \times \frac{1}{2} \times 50, \ldots \text{ 15 steps.}$$

i.e., $\frac{50}{8}, 2 \times \frac{50}{8}, 3 \times \frac{50}{8}, \ldots$ 15 steps.

Now, total volume of concrete,

$$V = \frac{50}{8} + 2 \times \frac{50}{8} + 3 \times \frac{50}{8} + \ldots = \frac{50}{8} [1 + 2 + 3 \ldots \text{ 15 terms}]$$

Note that the numbers in the bracket forms an AP with first term $(a) = 1$, common difference $(d) = 2 - 1 = 1$

and number of terms $(n) = 15$

$$\therefore \qquad V = \frac{50}{8} \times \frac{15}{2} [2 \times 1 + (15 - 1) \times 1]$$

$$\left[\because S_n = \frac{n}{2} \{2a + (n - 1)d\}\right]$$

$$= \frac{50}{8} \times \frac{15}{2} \times (2 + 14) = \frac{25 \times 15}{8} \times 16 = 750 \text{ m}^3$$

Hence, the total volume of concrete required to build terrace is 750 m^3.

43. (c) Given, initial money $P = ₹\ 2000$

Rate of interest, $R = 7\%$ per year; Time, $T = 1, 2, 3, 4, \ldots$

We know that, simple interest is given by the following formula

$$\text{SI} = \frac{PRT}{100}$$

$\therefore$ SI at the end of 1st yr $= \frac{2000 \times 7 \times 1}{100} = ₹\ 140$

SI at the end of 2nd yr $= \frac{2000 \times 7 \times 2}{100} = ₹\ 280$

SI at the end of 3rd yr $= \frac{2000 \times 7 \times 3}{100} = ₹\ 420$

Thus required list of numbers is 140, 280, 420, ...

Here, $280 - 140 = 420 - 280 \ldots = 140$

So, above list of numbers forms an AP, whose first term $(a) = 140$ and common difference $(d) = 140$.

Now, SI at the end of 20th yr will be equal to 20th term of the above AP.

$$\because \qquad a_{20} = a + (20 - 1)d = 140 + 19 \times 140$$

$$= 140 + 2660 = 2800$$

Hence, the interest at the end of 20th yr will be ₹ 2800.

44. (d) We have, $(\sec^2 A - 1) + \left(1 + \dfrac{1}{\tan^2 A}\right) = \tan^2 A + (1 + \cot^2 A)$ $\quad [\because \sec^2 A - 1 = \tan^2 A]$

$$= \tan^2 A + \text{cosec}^2 A = \frac{\sin^2 A}{\cos^2 A} + \frac{1}{\sin^2 A} \quad [\because 1 + \cot^2 A = \text{cosec}^2 A]$$

$$= \frac{\sin^4 A + \cos^2 A}{\sin^2 A(1 - \sin^2 A)} = \frac{\sin^4 A - \sin^2 A + 1}{\sin^2 A - \sin^4 A} \quad [\because \sin^2 A + \cos^2 A = 1]$$

$$= \frac{\sin^2 A - \sin^4 A\left(-1 + \dfrac{1}{\sin^2 A - \sin^4 A}\right)}{\sin^2 A - \sin^4 A}$$

$$= -1 + \frac{1}{\sin^2 A - \sin^4 A}$$

45. (d) Let $\quad f(x) = x^4 + 4x^3 + nx^2 + 4x + 1$

$$= (x^4 + 1) + 4x(x^2 + 1) + nx^2$$

$$= x^2\left[\left(x^2 + \frac{1}{x^2}\right) + 4\left(x + \frac{1}{x}\right) + n\right]$$

$$= x^2\left[\left(x + \frac{1}{x}\right)^2 - 2 + 4\left(x + \frac{1}{x}\right) + n\right]$$

$$= x^2\left[\left(x + \frac{1}{x}\right)^2 + 4\left(x + \frac{1}{x}\right) + (n - 2)\right]$$

$$= x^2\left[\left\{\left(x + \frac{1}{x}\right) + 2\right\}^2 + (n - 6)\right]$$

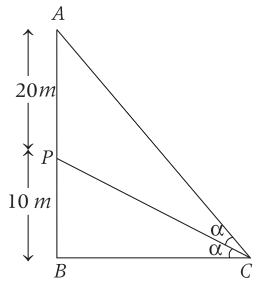
Here, $(n - 6)$ must be equal to zero, then $f(x)$ becomes a perfect square.

i.e., $n - 6 = 0 \Rightarrow n = 6$

46. (a) Let BA is a building and observer stands on point C.

In ΔPBC, $\quad \dfrac{PB}{BC} = \tan \alpha$

$\Rightarrow \quad \dfrac{10}{BC} = \tan \alpha \quad$ ...(i)



Now, in ΔABC, $\frac{AB}{BC} = \tan 2\alpha$

i.e., $\frac{30}{BC} = \frac{2\tan\alpha}{1-\tan^2\alpha}$ $\left[\because \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}\right]$

$$= \frac{2\left(\frac{10}{BC}\right)}{1-\frac{100}{(BC)^2}}$$ [$\because$ from Eq. (i)]

$\Rightarrow$ $\frac{30}{BC} = \frac{20\times BC}{BC^2-100}$

$\Rightarrow$ $30BC^2 - 3000 = 20BC^2 \Rightarrow 10BC^2 = 3000$

$\Rightarrow$ $BC^2 = 300$

$\therefore$ $BC = 10\sqrt{3}$ [taking positive square root]

$= 10(1.732) = 17.32$ m

47. (b) (i) Given, $XY \parallel AC$

$\therefore$ $\angle BAC = \angle BXY$ and $\angle BCA = \angle BYX$ [$\because$ corresponding angles]

By AAA similarity, $\Delta ABC \sim \Delta XBY$

By using the theorem, the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$\therefore$ $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta XBY)} = \frac{AB^2}{XB^2}$ ...(i)

Given, $\text{ar}(\Delta ABC) = 2\times \text{ar}(\Delta XBY)$

$\therefore$ $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta XBY)} = 2$ ...(ii)

From Eq. (i) and (ii); $\frac{AB^2}{XB^2} = 2 \Rightarrow \frac{AB}{XB} = \sqrt{2}$ [taking positive square root]

$AB = \sqrt{2}\times XB = \sqrt{2}(AB - AX)$

$\Rightarrow$ $\sqrt{2}AX = (\sqrt{2}-1)AB$

$\Rightarrow$ $\frac{AX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{(2-\sqrt{2})}{2}$

$\therefore$ $AX : AB = (2-\sqrt{2}) : 2$

(ii) Since, $\Delta ABC \sim \Delta XBY$

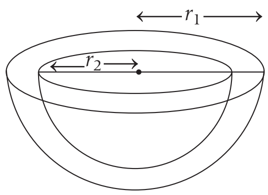
$$\therefore \quad \frac{AC}{XY} = \frac{AB}{BX}$$

$$= \frac{AB}{AB - AX} = \frac{1}{1 - \dfrac{AX}{AB}}$$

$$= \frac{1}{1 - \dfrac{2-\sqrt{2}}{2}} = \frac{2}{2 - 2 + \sqrt{2}}$$

$$= \frac{2}{\sqrt{2}} = \sqrt{2}$$

48. (d) External radius of hemispherical vessel, $r_1 = \dfrac{25}{2}$ cm



Internal radius of hemispherical vessel, $r_2 = \dfrac{24}{2} = 12$ cm

External curved surface area of hemispherical vessel $= 2\pi r_1^2 = 2 \times \dfrac{22}{7} \times \left(\dfrac{25}{2}\right)^2 = \dfrac{6875}{7}$ cm^2

Internal curved surface area of hemispherical vessel $= 2\pi r_2^2$

$$= 2 \times \frac{22}{7} \times 12^2 = \frac{6336}{7} \text{ cm}^2$$

Area of top of the hemispherical vessel

$$= \pi r_1^2 - \pi r_2^2 = \pi\left[\left(\frac{25}{2}\right)^2 - 12^2\right]$$

$$= \frac{22}{7}\left[\frac{625 - 576}{4}\right]$$

$$= \frac{22}{7} \times \frac{49}{4} = 38.5 \text{ cm}^2$$

Total surface area of the vessel

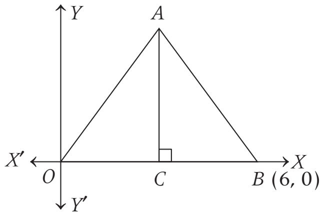
$$= \frac{6875}{7} + \frac{6336}{7} + 38.5$$

$$= 982.14 + 905.14 + 38.5$$

$$= 1925.78 \text{ cm}^2$$

Cost of painting the vessel at the rate of ₹ 0.5 per cm^2 $= 1925.78 \times 0.5 =$ ₹ 962.89

49. (c) Area of an equilateral triangle $= \dfrac{\sqrt{3}}{4}(\text{side})^2 = 9\sqrt{3}$



$$\Rightarrow \qquad \frac{\sqrt{3}}{4}(\text{side})^2 = 9\sqrt{3}$$

$$\Rightarrow \qquad (\text{side})^2 = 36$$

$$\Rightarrow \qquad \text{side} = \sqrt{36} \Rightarrow \text{side} = 6$$

Length of altitude $AC = \frac{\sqrt{3}}{2}(\text{Side}) = \frac{\sqrt{3}}{2} \times 6 = 3\sqrt{3}$

$= y$-coordinate of point A

Since, C is the mid-point of OB.

$\therefore$ Coordinate point of $C = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{6+0}{2}, \frac{0+0}{2}\right) = (3, 0)$

So, the coordinate of point $A = (3, 3\sqrt{3})$

50. (a) (i) In ΔRDC and ΔRAB,

$CD \| AB$ [$\because$ given]

$\angle BAR = \angle RDC$ [$\because$ Alternate interior angles]

$\angle ABR = \angle RCD$ [$\because$ Alternate interior angles]

$\therefore$ By AA similarity, $\Delta RDC \sim \Delta RAB$

$$\Rightarrow \qquad \frac{BR}{RC} = \frac{AB}{CD} \Rightarrow \frac{8.5}{1.5} = \frac{AB}{1.8}$$

$$\Rightarrow \qquad AB = \frac{8.5 \times 1.8}{1.5}$$

$$\Rightarrow \qquad AB = 10.2 \text{ m}$$

$\therefore$ Distance between the parts through town is 10.2 m.

(ii) In ΔRCD, $RD^2 = (CD)^2 - (RC)^2 = (1.8)^2 - (1.5)^2 = 3.24 - 2.25 = 0.99$

and $\frac{BR}{RC} = \frac{AR}{RD} \Rightarrow AR = \frac{8.5 \times 0.99}{1.5} = 5.61 \text{ m}$

$\therefore$ Required distance $= AR + RB = 5.61 + 8.5 = 14.11$ m