BLOOM | BMO Bloom Maths Olympiad

GRADE 9

Bloom Maths Olympiad
Sample Paper 1

Maximum Time : 60 Minutes   Maximum Marks : 60

INSTRUCTIONS

1. There are 50 Multiple Choice Questions in this paper divided into two sections :

   Section A 40 MCQs; 1 Mark each

   Section B 10 MCQs; 2 Marks each

2. Each question has Four Options out of which **ONLY ONE** is correct.
3. All questions are compulsory.
4. There is no negative marking.
5. No electronic device capable of storing and displaying visual information such as calculator and mobile is allowed during the course of the exam.

School Name

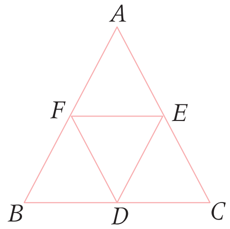
Student's Name

Section A (1 Mark Questions)

1. If $\frac{x}{y} + \frac{y}{x} = -1$, then $x^2(x^3 - y^3) = ?$

(a) –1 (b) 0 (c) 1 (d) 3

2. In the given figure, if area $(\Delta ABC) = 32 \text{cm}^2$, then area $(AEDF)$ is



(a) 18 cm^2 (b) 16 cm^2 (c) 20 cm^2 (d) 8 cm^2

3. What is the length of the largest pole that can be placed in a room 11 m long, 8 m broad and 9 m high?

(a) 33 m (b) 32 m (c) $2\sqrt{66.5}$ m (d) 23 m

4. If the mean of the following data is 18.75, then find the value of p.

x_i	10	15	p	25	30
f_1	5	10	7	8	2

(a) 20 (b) 10 (c) 15 (d) 17

5. The graph of $y = 5x$ is a line

(a) parallel to Y-axis
(b) parallel to X-axis
(c) passing through origin
(d) perpendicular to the Y-axis

6. What is the degree of the polynomial

$$4x^3 + 2x^2 + 5?$$

(a) 2 (b) 1 (c) 5 (d) 3

7. If a and b are zeroes of $x^2 - 6x + k$.

What is the value of k if $3a + 2b = 20$?

(a) 112 (b) –112 (c) 118 (d) –16

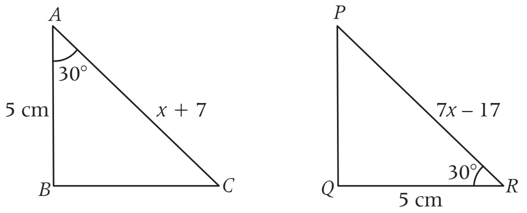
8. If $(5)^{x-3} \times (3)^{2x-8} = 225$, then find the value of x.

(a) 5 (b) 7 (c) 3 (d) 2

9. The perpendicular distance of point (– 21, – 4) from Y-axis will be

(a) 25 (b) 19 (c) 21 (d) 13

10. In the given triangles



What is the value of x?

(a) 5 (b) 4 (c) 3 (d) 2

11. If $x - \frac{1}{x} = 2$, then find $x^4 + \frac{1}{x^4}$.

(a) 30 (b) 4 (c) 9 (d) 34

12. The point (a, a) always lie on ....

(a) $x - y = 0$ (b) $x + y = 0$ (c) $x = +a$ (d) $y = +a$

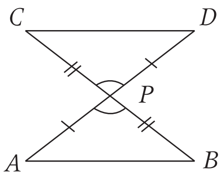
13. A cube of edge 'a' is divided into 'n' equal cubes. What will be the edge of the new cube?

(a) $\sqrt{na}$ (b) $\frac{a}{n}$ (c) $\sqrt[3]{na}$ (d) $\frac{a}{\sqrt[3]{n}}$

14. If the perpendicular distance of a point P from the X-axis is 5 units and the foot of the perpendicular lies on the negative direction of X-axis, then the point P has

(a) x-coordinate $= -5$ (b) y-coordinate $= 5$ only
(c) y-coordinate $= -5$ only (d) y-coordinate $= 5$ or -5

15. In the given figure, if $PA = PD$ and $PB = PC$. Then ΔPAB is congruent to



(a) ΔPDC by SAS (b) ΔCPD by ASA (c) ΔPCD by SAS (d) ΔDPC by ASA

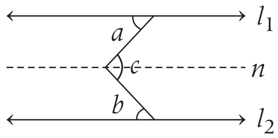
16. If $\sqrt{3} = 1.73205$, what is the value of $\sqrt{\frac{2(\sqrt{3}-1)}{\sqrt{3}+1}}$?

(a) 0.73205 (b) 2.73205 (c) 1.73205 (d) -2.0735

17. A bag contains 3 red balls and some green balls. If the probability of drawing a green ball is double that of a red ball, find the number of balls in the bag.

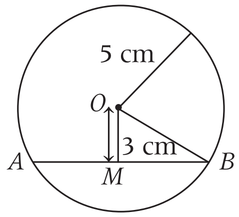
(a) 10 (b) 15 (c) 20 (d) 9

18. In the given figure, find the value of c, if a is two-third of b, which is a complement of 45°.



(a) 75° (b) 70° (c) 120° (d) 160°

19. In the given figure, O is the centre of a circle having radius = 5 cm. If $OM \perp AB$ and $OM = 3$ cm, then find the length of the chord AB.

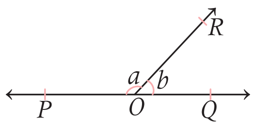


(a) 4 cm (b) 8 cm (c) 10 cm (d) 16 cm

20. The probability of guessing the correct answer to a certain question is $\frac{x}{5}$. If the probability of not guessing the correct answer is $\frac{2x}{3}$, then find the value of x.

(a) $\frac{1}{15}$ (b) $\frac{1}{13}$ (c) $\frac{13}{15}$ (d) $\frac{15}{13}$

21. In the given figure, if $\angle POR$ and $\angle QOR$ form a linear pair and $\angle a - \angle b = 80°$, then find the values of $\angle a$ and $\angle b$, respectively.



(a) 30° and 150° (b) 60° and 120° (c) 120° and 60° (d) 130° and 50°

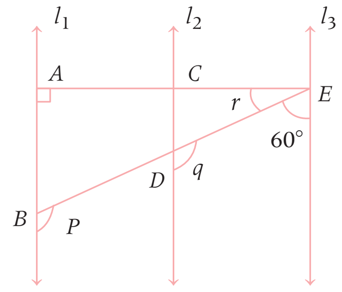
22. Write T for True and F for false.

I. Chord of a circle, which is twice as long as its radius is diameter.
II. There are two circles passing through three given non-collinear points.
III. A continuous piece of a circle is arc of the circle.
IV. Equal chords of a circle subtend equal angles at the centre.

Codes

	I	II	III	IV		I	II	III	IV
(a)	T	T	F	F	(b)	F	F	T	T
(c)	T	F	T	T	(d)	F	T	T	F

23. In the given figure, if $l_1 \parallel l_2 \parallel l_3$.



Find the value of $p + r$.

(a) 160° (b) 100° (c) 150° (d) 90°

24. If $81^x = \dfrac{9}{3^x}$, then find the value of x.

(a) $\dfrac{1}{5}$ (b) 3/5 (c) 2/5 (d) $\dfrac{3}{7}$

25. Consider the following statements and choose the correct option.

Statement 1 Volume of a cylinder is equal to the 3 times of the volume of a cone if these radii and heights are equal.

Statement 2 A cone is generated when a right angled triangle is rotated about its base.

(a) Both Statement 1 and 2 are true.
(b) Statement 1 is true and Statement 2 is false.
(c) Statement 1 is false and Statement 2 is true.
(d) Both statements are false.

26. Two vertices of an equilateral triangle are origin and (4, 0). What is the area of the triangle?

(a) 4 sq unit (b) $4\sqrt{3}$ sq units (c) $\sqrt{3}$ sq units (d) $\dfrac{\sqrt{3}}{4}$ sq units

27. What is the height of a solid cylinder of radius 5 cm and total surface area is 660 sq cm?

(a) 10 cm (b) 12 cm (c) 15 cm (d) 16 cm

28. If $\dfrac{(\sqrt{5}+1)}{(\sqrt{5}-1)} = a + b\sqrt{5}$.

Then, find the values of a and b.

(a) 1 and 2 (b) 2 and 1
(c) $\dfrac{3}{2}$ and $\dfrac{1}{2}$ (d) $\dfrac{1}{2}$ and 1

29. The point $(-5, 3)$ lie in which quadrant?

(a) Ist quadrant (b) IInd quadrant
(c) IIIrd quadrant (d) IVth quadrant

5

30. The parking charges of a car in a parking lot is ₹ 30 for the first 2 h and ₹ 10/h for subsequent hours. The above statement can be expressed in a linear equation as (if total parking time is x h and total charges is ₹ y).

(a) $5x - y + 5 = 0$ (b) $7x - y + 7 = 0$ (c) $10x - y + 10 = 0$ (d) $9x - y + 9 = 0$

31. Factorise $\frac{x^3}{8} - 64 - 3x^2 + 24x$ by using suitable identity and choose the correct option.

(a) $\left(\frac{x}{2} - 4\right)\left(\frac{x}{2} - 4\right)\left(\frac{x}{2} - 4\right)$ (b) $(x - 4)(x - 4)(x - 3)$

(c) $(x - 2)(x - 1)(x - 4)$ (d) None of these

32. Teachers and students are selected at random to make two teams of 30 members each on sport day to participate in the event of 'Tug of War'. The numbers of volunteers are as follows

Teachers		Students	
Male	**Female**	**Male**	**Female**
12	18	20	10

Find the probability that the person chosen at random is a female student

(a) $\frac{1}{5}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) $\frac{2}{3}$

33. If $a = \frac{2^{x-1}}{2^{x-2}}$, $b = \frac{2^{-x}}{2^{x+1}}$ and $a - b = 0$, find the value of x.

(a) -1 (b) -2 (c) 3 (d) 0

34. If $x = 3$ and $y = 1$, find the value of $(x^y + y^x)^{-1}$.

(a) 1/2 (b) $\frac{1}{4}$ (c) 4 (d) 2

35. The graph of the linear equation $7x + y = 9$ is a line which meets the X-axis at the point …… .

(a) $\left(\frac{9}{7}, 0\right)$ (b) (0, 9) (c) $\left(\frac{9}{7}, 9\right)$ (d) (0, 7)

36. An equilateral ΔTQR is drawn inside a square $PQRS$. The value of $\angle PTS$ is

(a) 75° (b) 90° (c) 120° (d) 150°

37. In a circle of radius 3 units, a diameter AB intersects a chord of length 2 units perpendicular at P. If $AP > BP$, then what is the ratio of AP to BP?

(a) $(3 + \sqrt{10}) : (3 - \sqrt{10})$ (b) $(3 + \sqrt{8}) : (3 - \sqrt{8})$

(c) $(3 + \sqrt{3}) : (3 - \sqrt{3})$ (d) $3 : \sqrt{3}$

38. A single 6-sided die is rolled. What is the probability of getting either (1 or 5)?

(a) $\frac{1}{6}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

39. Which quadrilateral is formed by joining the points (1, 1) (2, 4) (8, 4) and (10, 1)?

(a) Rectangle (b) Trapezium
(c) Square (d) Triangle

40. If the mean of 10 observations $x_1, x_2, x_3, ... x_{10}$ is $\bar{x}$, find the mean of the observations.

$\frac{x_1}{6}, \frac{x_2}{6}, \frac{x_3}{6}, \frac{x_4}{6}, ... \frac{x_{10}}{6}$.

(a) $\sqrt{6}\,\bar{x}$ (b) $6\bar{x}$ (c) $\frac{\bar{x}}{6}$ (d) $60\,\bar{x}$

Section B (2 Marks Questions)

41. If x, y and z are positive real numbers, then find the value of $\sqrt{x^{-1}y} \cdot \sqrt{y^{-1}z} \cdot \sqrt{z^{-1}x}$.

(a) 0 (b) 1
(c) –1 (d) Not defined

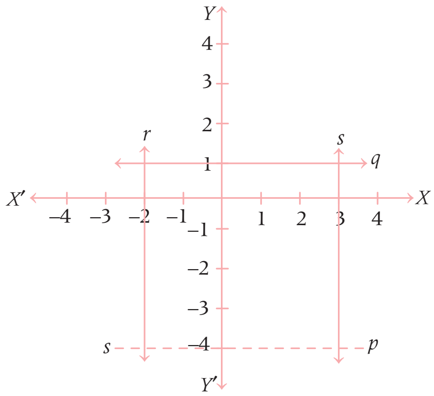
42. Match the linear equations in Column I with their solutions in Column II

	Column I		Column II
A.	$2x + 3y = 12$	(i)	(2, 3)
B.	$\frac{x-2}{3} = y - 3$	(ii)	(3, 2)
C.	$\frac{x}{2} - \frac{y}{3} = 2$	(iii)	(4, 4)
D.	$x - y = 0$	(iv)	(2, –3)

Codes

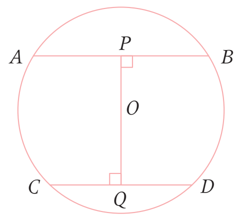
	A	B	C	D
(a)	ii	i	iv	iii
(b)	iii	i	ii	iv
(c)	i	ii	iii	iv
(d)	iv	iii	i	ii

43. Find the area enclosed by the lines p, q, r and s.



(a) 36 sq units (b) 49 sq units (c) 20 sq units (d) 25 sq units

44. In the given figure, *AB* and *CD* are two parallel chords of a circle with centre *O* and radius 5 cm. Also, $AB = 8$ cm and $CD = 6$ cm. If $OP \perp AB$ and $OQ \perp CD$, then determine the length of *PQ*.

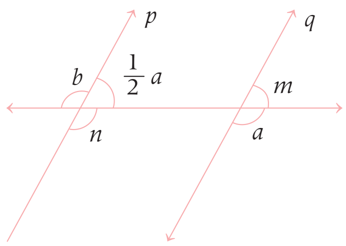


(a) 7 cm (b) 10 cm (c) 8 cm (d) None of these

45. From the points $A(2, 0)$, $B(2, 2)$, $C(0, 2)$ and after joining the line *OA*, *AB*, *BC* and *CO*, which figure do you obtain? (Here *O* is origin)

(a) Rectangle
(b) Square
(c) Parallelogram
(d) None of these

46. In the given figure, if $p \| q$, what is the value of *b*?



(a) 220° (b) 160° (c) 120° (d) 100°

47. The remainder of the polynomial $5 + bx - 2x^2 + ax^3$, when divided by $(x - 2)$ is twice the remainder when it is divided by $(x + 1)$.

Then, find the value of $10a + 4b$.

(a) 8
(b) 9
(c) 7
(d) 2

48. Three coins are tossed simultaneously 150 times with the following frequencies of different outcomes.

Number of tails	0	1	2	3
Frequency	25	30	32	63

Compute the probability of getting

(i) atleast 2 tails (ii) exactly 1 tail.

(a) $\frac{19}{30}, \frac{1}{5}$

(b) $\frac{1}{5}, \frac{1}{30}$

(c) $\frac{1}{30}, \frac{1}{19}$

(d) $\frac{3}{4}, \frac{1}{7}$

49. Let $ABCD$ be a rectangle and P, Q, R, S be the mid-points of sides AB, BC, CD, DA respectively. Then, the quadrilateral $PQRS$ is a

(a) square

(b) rectangle but need not be a square

(c) rhombus but need not be a square

(d) parallelogram but need not be a rhombus

50. The area formed by the line $3 = x + 2y$ with the coordinate axes is

(a) 3 sq units

(b) 2 sq units

(c) 2.25 sq units

(d) None of these

Solutions

1. (b) According to the question, $\frac{x}{y}+\frac{y}{x}=-1$

$$x^2+y^2=-xy$$

$$x^2+y^2+xy=0 \quad \text{... (i)}$$

As, $(x^3-y^3)=(x-y)(x^2+y^2+xy)$ (using identity) ... (ii)

From Eq. (i) and (ii)

$$(x^3-y^3)=(x-y)(0)=0$$

$$\therefore \quad x^2(x^3-y^3)=0$$

2. (b) From the given figure, D, E and F are the mid-points of sides BC, AC and AB.

Then, area of (ΔABC) = area of $[\Delta AEF+\Delta FED+\Delta EDC+\Delta FBD]$

Also, area of ΔAEF = area of ΔFED = area of ΔEDC = area of ΔFBD

$\Rightarrow$ area of $(\Delta AFE)=\frac{\text{area of }(\Delta ABC)}{4}=\frac{32}{4}=8\text{ cm}^2$

Now, area of parallelogram $AEDF$ = area of $(\Delta AEF+\Delta EFD)=(8+8)\text{ cm}^2=16\text{ cm}^2$

3. (c) Here, $l=11$ m, $b=8$ m and $h=9$ m

$\therefore$ Length of the largest pole = Length of the diagonal of room

$$=\sqrt{l^2+b^2+h^2}=\sqrt{11^2+8^2+9^2}$$

$$=\sqrt{121+64+81}=\sqrt{266}$$

$$=\sqrt{4\times 66.5}=2\sqrt{66.5}\text{ m}$$

4. (a) Frequency distribution table is given below

x_i	f_i	$f_i x_i$
10	5	50
15	10	150
p	7	$7p$
25	8	200
30	2	60
Total	$\Sigma f_i=32$	$\Sigma f_i x_i=(460+7p)$

$$\therefore \quad \text{Mean}=\frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\Rightarrow \quad \frac{460+7p}{32}=18.75 \quad \Rightarrow \quad 460+7p=600$$

$\Rightarrow \quad 7p = 140 \quad \Rightarrow \quad p = \dfrac{140}{7}$

$\therefore \quad p = 20$

5. (c) Graph of line $y = 5x$ always passes through origin.

6. (d) The given polynomial is $4x^3 + 2x^2 + 5$

The highest power of the variable x is 3.

So, the degree of the polynomial is 3.

7. (d) Here, $a + b = \dfrac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\left(\dfrac{-6}{1}\right) = 6$ ... (i)

and $ab = \dfrac{\text{Constant}}{\text{Coefficient of } x^2} = \dfrac{k}{1} = k$ ... (ii)

According to the question,

$$3a + 2b = 20 \qquad \text{... (iii)}$$

Multiplying Eq. (i) by 3 and subtracting Eq. (ii),

$$\begin{array}{r} 3a + 3b = 18 \\ \underline{\underset{-}{3a} \underset{-}{+} 2b \underset{-}{=} 20} \\ \boxed{b = -2} \end{array}$$

Putting the value of b in Eq. (i)

$$a - 2 = 6$$
$$a = 8$$

Now, from Eq. (ii) $\quad ab = k$

$\Rightarrow \quad k = (8)(-2) = -16$

8. (a) According to the question,

$$(5)^{x-3} \times (3)^{2x-8} = 225 \Rightarrow (5)^{x-3} \times (3)^{2x-8} = 3^2 \times 5^2$$

Comparing the power of both sides,

$$x - 3 = 2 \text{ or } 2x - 8 = 2$$

$\therefore \quad x = 5; \text{ or } 2x = 2 + 8 = x = \dfrac{10}{2} = 5$

Hence, $\quad x = 5$

9. (c) Perpendicular distance of point $(-21, -4)$ from Y-axis = |abscissa| = $|-21| = 21$

10. (b) In ΔBAC and ΔQRP,

$$\angle A = \angle R = 30^\circ$$

$$BA = QR = 5 \text{ cm}$$

$$\angle ABC = \angle RQP = 90^\circ$$

$$\Delta ABC \cong \Delta RQP$$ [by ASA criterion]

$$AC = RP$$ [CPCT]

$$x + 7 = 7x - 17$$

$\Rightarrow$ $$6x = 24 \Rightarrow x = 4$$

11. (d) $\left(x - \frac{1}{x}\right) = 2$

Squaring both sides, we get

$$x^2 + \frac{1}{x^2} - 2x \times \frac{1}{x} = 4 \quad \Rightarrow \quad x^2 + \frac{1}{x^2} = 6$$

Again, squaring both sides

$$(x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2x^2.\left(\frac{1}{x^2}\right) = 36$$

$\Rightarrow$ $$x^4 + \frac{1}{x^4} = 36 - 2$$

$\Rightarrow$ $$x^4 + \frac{1}{x^4} = 34$$

12. (a) Given point (a, a)

Here, we see that both coordinates has same value.

$\therefore$ $$x = y \Rightarrow x - y = 0$$

So, point (a, a) always lie on $x - y = 0$.

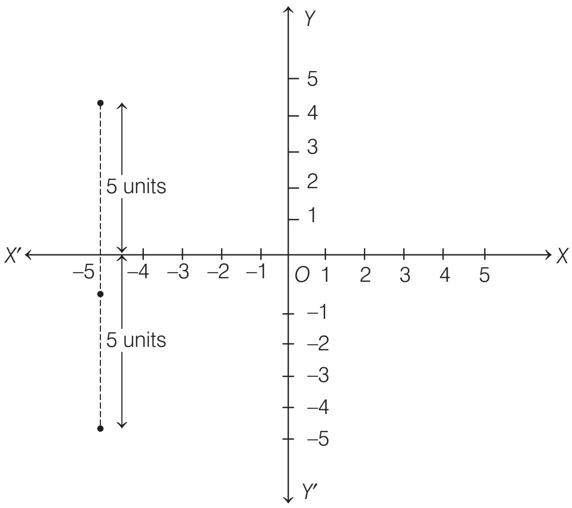
13. (d) Let the edge of new cube be a_1.

Volume of the bigger cube = Sum of the volume of n smaller cubes

$\Rightarrow$ $$a^3 = na_1^3$$ [$\because$ Volume of Cube = $(\text{side})^3$]

$\therefore$ $$a_1 = \frac{a}{\sqrt[3]{n}}$$

14. (d) According to the given condition;



Hence, the point P has y-coordinate $= 5$ or -5.

15. (a) Consider ΔPAB and ΔPDC,

$\angle BPA = \angle CPD$ [vertically opposite angles are equal]

$PB = PC$ [given]

and $PA = PD$ [given]

$\therefore$ $\Delta PAB \cong \Delta PDC$ [by SAS congruence rule]

16. (a) $\sqrt{\dfrac{2(\sqrt{3}-1)}{\sqrt{3}+1}} = \sqrt{\dfrac{2(\sqrt{3}-1)}{(\sqrt{3}+1)} \times \dfrac{(\sqrt{3}-1)}{(\sqrt{3}-1)}}$

$= \sqrt{\dfrac{2(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2}} = \sqrt{\dfrac{2(\sqrt{3}-1)^2}{2}}$ $[\because (a+b)(a-b) = a^2 - b^2]$

$= \sqrt{(\sqrt{3}-1)^2} = \sqrt{3} - 1 = 1.73205 - 1 = 0.73205$

17. (d) Let the total numbers of balls are n.

Probability of drawing a red ball $= \dfrac{3}{n}$

Probability of drawing a green ball $= \dfrac{6}{n}$

We know that, sum of probability of all outcomes $= 1$

Then, $\dfrac{3}{n} + \dfrac{6}{n} = 1$

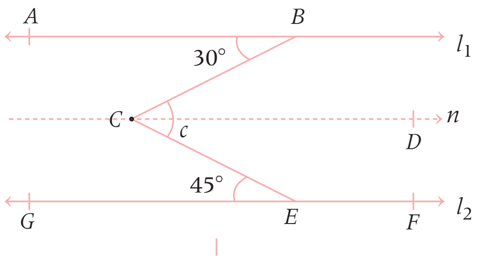
$\dfrac{9}{n} = 1$

$\Rightarrow$ $n = 9$

$\therefore$ Total number of balls $= 9$

18. (a) b is a complement of $45° = 90° - 45° = 45°$

and $\quad a = \frac{2}{3}b \Rightarrow a = \frac{2}{3} \times 45° = 30°$



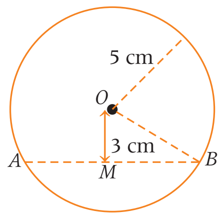
Since, line n is parallel to l_1 and l_2.

$$\angle DCB = \angle ABC = 30°$$

and $\quad \angle DCE = \angle CEG = 45°$ [alternate angles]

From given figure, $c = \angle DCE + \angle DCB = 45° + 30° = 75°$

19. (b) Given, $OM \perp AB \Rightarrow M$ is the mid-point of AB.



Join OB

Now, in ΔOMB

$$MB^2 = OB^2 - OM^2 \quad \text{[using Pythagoras theorem]}$$

$$= (5)^2 - (3)^2 = 25 - 9$$

$$MB = \sqrt{16} = 4 \text{ cm}$$

$\therefore \quad AB = 2 \times MB = (4 \times 2) = 8$ cm [$\because$ perpendicular line drawn from centre, of circle to the chord, it divides the chord equally]

20. (d) $P(A) + P(\overline{A}) = 1 \Rightarrow \frac{x}{5} + \frac{2x}{3} = 1$

$\Rightarrow \quad \frac{3x + 10x}{15} = 1 \Rightarrow 13x = 15$

$\therefore \quad x = \frac{15}{13}$

21. (d) By linear pairs $\angle a + \angle b = 180°$ ... (i)

Given $\quad \angle a - \angle b = 80°$ [given] ... (ii)

On solving Eq. (i) and (ii), we get the values

$$\angle a = 130°, \angle b = 50°$$

22. (c) Only IInd statement is false.

As there is one and only one circle passing through three given non-collinear points.

23. (c) Since $l_1 \parallel l_3$, therefore the sum of interior angles is 180°.

$$p + 60^\circ = 180^\circ \quad [\because l_1 \parallel l_3]$$

$$p = 180^\circ - 60^\circ = 120^\circ$$

and $\quad r = 90^\circ - 60^\circ = 30^\circ$

$\therefore \quad p + r = 120^\circ + 30^\circ = 150^\circ$

24. (c) According to the question,

$$81^x = \frac{9}{3^x}$$

$$(3^4)^x = \frac{3^2}{3^x}$$

$\Rightarrow \quad 3^{4x} = 3^{2-x}$

Comparing the power of both sides, we get

$$4x = 2 - x$$

$\Rightarrow \quad 5x = 2 \Rightarrow x = \frac{2}{5}$

25. (a) Statement 1 is true.

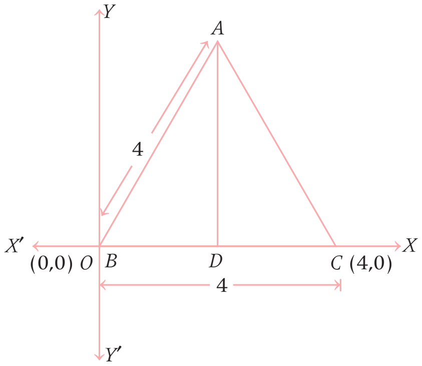
As, volume of cylinder $= \pi r^2 h$

$$= 3 \times \frac{1}{3}\pi r^2 h = 3 \times \text{Volume of a cone.}$$

Statement 2 is also true.

26. (b) Since, triangle is an equilateral.

$\therefore \quad AB = BC = CA = 4$



Area of an equilateral triangle $= \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4} \times (4)^2 = 4\sqrt{3}$ sq units.

27. (d) Let the height of solid cylinder be h.

Given, radius $(r) = 5$ cm

Total surface area of cylinder $= 660 \text{ cm}^2$ [given]

$\Rightarrow \quad 2\pi rh + 2\pi r^2 = 660 \Rightarrow 2\pi r(h + r) = 660$

$$\Rightarrow \quad (h+5)=\frac{330}{5\pi}=\frac{330}{5}\times\frac{7}{22} \Rightarrow h+5=\frac{66\times 7}{22}$$

$$\Rightarrow \quad h=21-5=16 \text{ cm}$$

28. (c) According to the question,

$$\frac{\sqrt{5}+1}{\sqrt{5}-1}=a+b\sqrt{5}$$

$$\frac{\sqrt{5}+1}{\sqrt{5}-1}\times\frac{\sqrt{5}+1}{\sqrt{5}+1}=a+b\sqrt{5} \qquad \text{[by rationalisation]}$$

$$\Rightarrow \quad \frac{(\sqrt{5}+1)^2}{5-1}=a+b\sqrt{5} \Rightarrow \frac{5+1+2\sqrt{5}}{4}=a+b\sqrt{5} \qquad [\because (a+b)$$

$(a-b)=a^2-b^2]$

$$\Rightarrow \quad \frac{6}{4}+\frac{2\sqrt{5}}{4}=a+b\sqrt{5} \Rightarrow \frac{3}{2}+\frac{1}{2}\sqrt{5}=a+b\sqrt{5}$$

By comparing LHS and RHS

$$a=\frac{3}{2} \text{ and } b=\frac{1}{2}$$

29. (b) In a point $(-5, 3)$, x-coordinate is negative and y-coordinate is positive,so it is lies in II quadrant.

30. (c) Given, parking charges for the first 2 h = ₹ 30

and for subsequent hours = ₹ 10

According to the given condition,

$$30+10(x-2)=y$$

$$\Rightarrow \quad 30+10x-20=y$$

$$\Rightarrow \quad 10x-y+10=0$$

which is the required linear equation in two variables.

31. (a) We have, $\frac{x^3}{8}-64-3x^2+24x=\left(\frac{x}{2}\right)^3-(4)^3-3x(x-8)$

$$=\left(\frac{x}{2}\right)^3-(4)^3-3\times\frac{x}{2}\times 4\left(\frac{x}{2}-4\right)=\left(\frac{x}{2}-4\right)^3 \qquad [\because a^3-b^3-3ab(a-b)=(a-b)^3]$$

$$=\left(\frac{x}{2}-4\right)\left(\frac{x}{2}-4\right)\left(\frac{x}{2}-4\right)$$

32. (b) Total number of students $=20+10=30$

Number of female students $=10$

$\therefore$ Probability that the person is a female student $= \frac{10}{30} = \frac{1}{3}$

33. (a) Given, $a = \frac{2^{x-1}}{2^{x-2}}$ and $b = \frac{2^{-x}}{2^{x+1}}$

According to the question,

$$a - b = 0$$

$$\Rightarrow \quad \frac{2^{x-1}}{2^{x-2}} - \frac{2^{-x}}{2^{x+1}} = 0$$

$$\Rightarrow \quad 2^{x-1-x+2} - 2^{-x-x-1} = 0 \qquad \left[\frac{a^m}{a^n} = a^{m-n}\right]$$

$$\Rightarrow \quad 2^1 - 2^{-2x-1} = 0$$

$$\Rightarrow \quad 2^{-2x-1} = 2^1$$

$$\Rightarrow \quad -2x - 1 = 1 \qquad \text{[compare the exponents]}$$

$$\Rightarrow \quad 2x = -2$$

$$\therefore \quad x = -1$$

34. (b) Given, $x = 3$ and $y = 1$

$$\therefore \quad (x^y + y^x)^{-1} = [(3)^1 + (1)^3]^{-1} = (3+1)^{-1} = (4)^{-1} = \frac{1}{4}$$

35. (a) We have, $7x + y = 9$

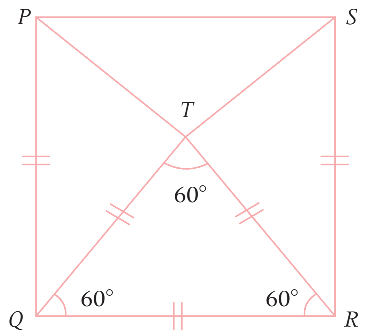
Since, the line meets X-axis

i.e. $y = 0$

Now, $7x + 0 = 9 \Rightarrow x = \frac{9}{7}$

$\therefore$ Required point is $\left(\frac{9}{7}, 0\right)$.

36. (d) In ΔSRT,



$$\angle SRT = 90° - 60° = 30° \qquad [\because \angle SRQ = 90°]$$

$$\angle RTS + \angle TSR + \angle SRT = 180°$$

$$2\angle RTS + \angle SRT = 180^\circ \qquad [\because \angle RTS = \angle TSR]$$

$$2\angle RTS + 30^\circ = 180^\circ$$

$$2\angle RTS = 180^\circ - 30^\circ$$

$\therefore$ $$\angle RTS = \frac{1}{2}(180^\circ - 30^\circ) = 75^\circ$$

Similarly, $\angle PTQ = 75^\circ$

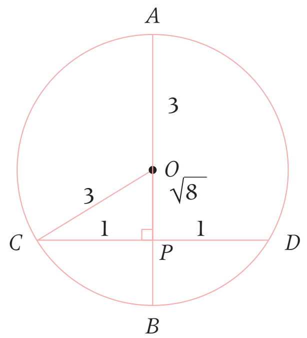
Since, sum of all angles around a point is 360°

$\therefore$ $$\angle PTS + \angle PTQ + \angle QTR + \angle RTS = 360^\circ$$

$\Rightarrow$ $$\angle PTS + 75^\circ + 60^\circ + 75^\circ = 360^\circ$$

$\Rightarrow$ $$\angle PTS = 360^\circ - 210^\circ = 150^\circ$$

37. (b) In ΔOPC, $OP = \sqrt{(3)^2 - (1)^2} = \sqrt{9-1} = \sqrt{8}$



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Now, $AP = AO + OP$

$= (3 + \sqrt{8})$ cm [$\because AO =$ radius]

Also, $BP = BO - OP = (3 - \sqrt{8})$ cm [$\because BO =$ radius]

$\therefore$ Required ratio $= AP : BP = (3 + \sqrt{8}) : (3 - \sqrt{8})$

38. (d) $\because$ Total outcomes when a die is rolled $= 6$

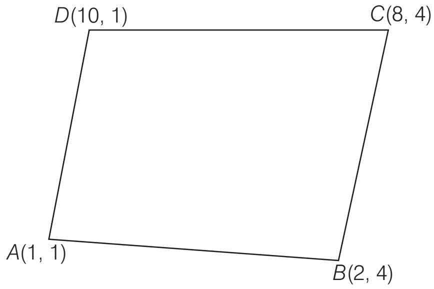
Probability of getting a number in a single throw of die $= \frac{1}{6}$

So, $P(1) = \frac{1}{6}$ and $P(5) = \frac{1}{6}$

$\therefore$ Required probability of getting either 1 or 5

$$= P(1) + P(5) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

39. (b) Let the points (1, 1), (2, 4), (8, 4) and (10, 1) formed a quadrilateral $ABCD$.



$\therefore$ $AB = \sqrt{(1-2)^2 + (1-4)^2} = \sqrt{1+9} = \sqrt{10}$

[$\because$ Distance formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$]

$BC = \sqrt{(2-8)^2 + (4-4)^2} = \sqrt{36+0} = \sqrt{36} = 6$

$CD = \sqrt{(10-8)^2 + (1-4)^2} = \sqrt{4+9} = \sqrt{13}$

$DA = \sqrt{(10-1)^2 + (1-1)^2} = \sqrt{81} = 9$

Here, all sides are different.

So, $ABCD$ is a trapezium.

40. (c) Since, the mean of 10 observations $x_1, x_2, x_3, \ldots, x_{10}$ is $\bar{x}$. ...(i)

Then, $\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_{10}}{10}$

Now mean of observations is

$$\frac{x_1}{6}, \frac{x_2}{6}, \frac{x_3}{6}, \ldots, \frac{x_{10}}{6}$$

$$= \frac{1}{10}\left(\frac{x_1}{6} + \frac{x_2}{6} + \ldots + \frac{x_{10}}{6}\right)$$

$$= \frac{1}{10}\frac{(x_1 + x_2 + x_3 + \ldots + x_{10})}{6}$$

$$= \frac{1}{6} \times \bar{x} \qquad \text{[from Eq. (i)]}$$

$$= \frac{\bar{x}}{6}$$

41. (b) $\sqrt{x^{-1}y} \cdot \sqrt{y^{-1}z} \cdot \sqrt{z^{-1}x} = \sqrt{\frac{y}{x}} \cdot \sqrt{\frac{z}{y}} \cdot \sqrt{\frac{x}{z}} = \left(\frac{y}{x}\right)^{1/2}\left(\frac{z}{y}\right)^{1/2}\left(\frac{x}{z}\right)^{1/2}$

$$= \frac{(y)^{1/2}}{(x)^{1/2}} \cdot \frac{(z)^{1/2}}{(y)^{1/2}} \cdot \frac{(x)^{1/2}}{(z)^{1/2}} = 1$$

42. (a) A. $2x + 3y = 12$

Putting $x = 3$, $y = 2$ in the equation, we get

$2 \times 3 + 3 \times 2 = 12 \Rightarrow 12 = 12$

$\therefore$ (3, 2) is the solution of the equation.

B. $\frac{x-2}{3} = y - 3$

Putting $x = 2$, $y = 3$ in the equation, we get

$\frac{2-2}{3} = 3 - 3 \Rightarrow 0 = 0$

$\therefore$ (2, 3) is the solution of the equation.

C. $\frac{x}{2} - \frac{y}{3} = 2$

Putting $x = 2$, $y = -3$ in the equation, we get

$\frac{2}{2} + \frac{3}{3} = 2 \Rightarrow 1 + 1 = 2 \Rightarrow 2 = 2$

$\therefore$ $(2, -3)$ is the solution of the equation.

D. $x - y = 0$

$\Rightarrow \qquad x = y$

$\therefore$ (4, 4) is the solution of the equation.

A $\to$ (ii), B $\to$ (i), C $\to$ (iv), D $\to$ (iii)

43. (d) The lines p, q, r and s form a square.

Side of square = 5 units

$\therefore$ Area of square $= (\text{side})^2 = 5 \times 5 = 25$ sq units

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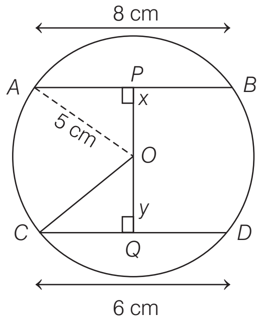
44. (a) Apply Pythagoras theorem in ΔAOP and ΔCOQ.



In ΔAOP, $AO^2 = AP^2 + PO^2$

$$\Rightarrow \quad AO^2 = \left(\frac{AB}{2}\right)^2 + PO^2$$

[$\because$ perpendicular from centre to the chord bisects the chord]

$$\Rightarrow \quad (5)^2 = (4)^2 + x^2$$

$$\Rightarrow \quad 25 - 16 = x^2$$

$$\therefore \quad x = \sqrt{9} = 3 \text{ cm}$$ [taking positive square root]

In ΔCOQ, $CO^2 = OQ^2 + CQ^2$

$$\Rightarrow \quad CO^2 = OQ^2 + \left(\frac{CD}{2}\right)^2 \qquad \left[\because CQ = \frac{CD}{2}\right]$$

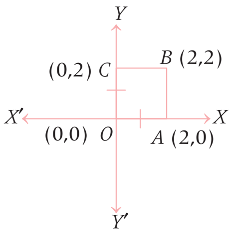
$$\Rightarrow \quad (5)^2 = y^2 + (3)^2$$

$$\Rightarrow \quad 25 - 9 = y^2$$

$$\Rightarrow \quad y = \sqrt{16} = 4 \text{ cm}$$ [taking positive square root]

$$\therefore \quad PQ = OP + OQ = 3 + 4 = 7 \text{ cm}$$

45. (b) When we plot the given points $A(2, 0), B(2, 2), C(0, 2)$ in the cartesian plane.

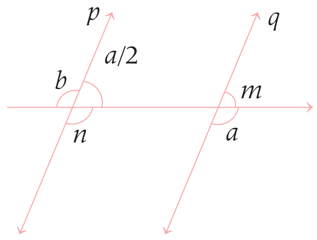


Now, join OA, AB, BC and CO.

We see that $OA = AB = BC = CO = 2$ units and A lies on the X-axis, C lies on the Y-axis and O is the origin, then each angle of $OABC$ is 90°.

$\therefore$ We obtain square $OABC$.

46. (c) Here, $\frac{a}{2} = m$ and $n = a$ [as corresponding angles are equal]



Then, $\frac{a}{2} + n = 180^\circ \Rightarrow \frac{1}{2}a + a = 180^\circ \Rightarrow a = 120^\circ$ [Linear pair axiom]

and $b = n$ [vertically opposite angles are equal]

$\therefore$ $b = 120^\circ$ [$\because n = a$]

47. (b) Let $f(x) = 5 + bx - 2x^2 + ax^3$

When $f(x)$ is divided by $(x-2)$, then the remainder is $f(2)$.

When $f(x)$ is divided by $(x+1)$, then the remainder is $f(-1)$.

Now, $f(2) = 5 + b(2) - 2(2)^2 + a(2)^3 = 8a + 2b - 3$

and $f(-1) = 5 + b(-1) - 2(-1)^2 + a(-1)^3 = -a - b + 3$

According to the question, $f(2) = 2f(-1)$

$\therefore$ $8a + 2b - 3 = 2(-a - b + 3)$

$\Rightarrow$ $8a + 2b - 3 = -2a - 2b + 6$

$\Rightarrow$ $10a + 4b = 9$

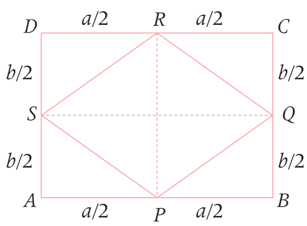
48. (a) (i) Number of atleast 2 tails $= 32 + 63 = 95$

Probability of getting atleast 2 tails $= \frac{95}{150} = \frac{19}{30}$

(ii) Probability of getting exactly 1 tail $= \frac{30}{150} = \frac{1}{5}$

49. (c) Given, $ABCD$ is a rectangle.

$\therefore \angle A = \angle B = \angle C = \angle D = 90^\circ$



Let $\quad AB = CD = a$

and $\quad BC = AD = b$

Also, given P, Q, R and S are mid-points of AB, BC, CD and AD, respectively.

Now, in ΔPBQ,

$$PQ^2 = PB^2 + QB^2 \qquad \text{[use pythagoras theorem]}$$

$$PQ^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2$$

$$\Rightarrow \quad PQ = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2} \qquad \text{[taking positive square root]}$$

Similarly, in ΔQCR, ΔRDS and ΔSAP,

$$PQ = RQ = SR = SP = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2}$$

$\therefore$ Quadrilateral $PQRS$ can be a square or rhombus but here the diagonals of $PQRS$ are not equal.

Hence, $PQRS$ is a rhombus.

50. (c) Given, equation of line is $3 = x + 2y$ ...(i)

Put $x = 0$ in Eq. (i), we get $3 = 0 + 2y$

$$\Rightarrow \quad y = \frac{3}{2}$$

Line meets the Y-axis at point $\left(0, \dfrac{3}{2}\right)$

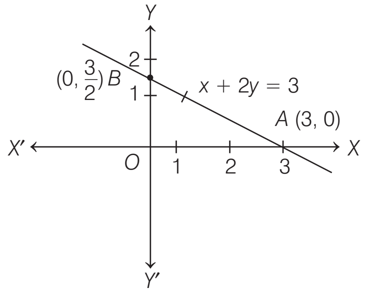
Put $y = 0$ in Eq. (i), we get

$$3 = x + 2 \times 0$$

$$\Rightarrow \quad x = 3$$

$\because$ Line meets the X-axis at point (3, 0)

The line $x + 2y = 3$ is shown on a graph paper as given below



From the graph, we see that line makes a triangle with the coordinate axes.

$\therefore$ Area of $\Delta OAB = \frac{1}{2} \times OA \times OB = \frac{1}{2} \times 3 \times \frac{3}{2} = \frac{9}{4} = 2.25$ sq unit

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